1. B	2. D	3. D	4. B	5. C
6. A	7. C	8. A	9. D	10. A
11. C	12. D	13. A	14. B	15. E: 9580
16. B	17. A	18. D	19. E: $y^2 - x^2 = 1$	20. B
21. C	22. D	23. C	24. B	25. B
26. D	27. B	28. D	29. D	30. B

1. **B:** By summing each of the digits and computing that remainder when divided by 9, we can compute the remainder when the full 10 digit number is divided by 9. 8+4+6+0+9+3+4+8+7+2=51. 51 leaves a remainder of 6 when divided by 9, therefore 8,460,934,872 will leave a remainder of 6 when divided by 9, A.

2. **D:** First, notice that the magnitude of $\sqrt{5} + 1 + i\left(\sqrt{10 - 2\sqrt{5}}\right)$ is easily calculated to be 4 by squaring the real and imaginary parts and summing them. $\sqrt{5} + 1 = 4\cos(36)$. Since our magnitude is 4, we know that the imaginary part must be $4\sin(36)$ from the relation $sin^2(x) + cos^2(x) = 1$. Now we can rewrite the equation as $x + yi = (4cis(36))^6$. $x + yi = 4096cis(216)$. We could try to calculate cos(216); however, note that 216 = 180+36, and since $\cos\theta = -\cos(180 + \theta)$. So $x = -4096\cos(36) = -1024(\sqrt{5} + 1)$, D.

3. **D:** From the Pythagorean theorem, we know that $a^2 + b^2 = c^2$. We are given a value for a, and since we know that $a^2 = c^2 - b^2$, we can rewrite our equation as $103^2 = 10{,}609 = (c + b)(c - b)$. Notice that each of our pairs have differences of either 1 or 2. Since 10609 is not even, we can immediately remove A and C as possible answers. Now, all we have to do is find the pair of numbers that sums to 10,609, which is 5,304 & 5,305, D.

4. **B:** We could expand this answer, but it is simpler to sum the coefficients of the original polynomial, and then take that number to the eighth power.$6 + 4 - 5 - 2 = 3$. Thus, the sum of the coefficients in the expanded form must be $3^8 = 6561$. B.

5. **C:** For any number ending in the digit 5, call it $n = 10a + 5$, that number's square can be written as $n^2 = 100(a)(a + 1) + 25$. Applying this to our problem, we just need to calculate 103x104. By splitting these numbers into 100+3 and 100+4, we can multiply quickly, getting $100^2 + 3 \times 100 + 4 \times 100 + 3 \times 4$, which can be easily evaluated into 10712. Thus our final answer is 1071225, C.

6. **A:** First, we can observe that the units digit of the cube is 4, meaning that the cube root must have a units digit of 4 (this can be shown by simply mapping each unit digit to the units digit of its square, which results in a 1-to-1 correspondence). Secondly, since our perfect cube is 6 digits, we know that the cube root must be a 2-digit number (100^3=1,000,000). Notice that $7^3=343<405<512=8^3$. This tells us that our tens digit is 7. Therefore, our cube root, which we know is an integer, must be 74, A.

7. **C:** Converting from base 2 into base 16 (hexadecimal), we can group the binary value into groups of 4 and convert those. $10\ 1110\ 1011\ 0011_2 = (2)(14)(13)(3)_{16}$. Note that any numbers 10 or greater are then converted into letters with A=10, B=11, etc. So our answer is 2EB3, C.

8. **A:** Instead of multiplying all of the matrices together, we can compute each determinant separately and then multiply all of them together since $\det(AB) = \det(A)\det(B)$. $\det(A) = -1, \det(B) = -1, \det(C) = 1, \det(D) = -1$. Thus, $\det(ADCB) = -1$, A.

9. **D:** The sum of the first n cubes can be expressed as $\frac{n^2(n+1)^2}{4}$. The sum of the first n squares can be expressed as $\frac{n(n+1)(2n+1)}{6}$. Breaking apart our expression, $\sum_{n=1}^{5}(4n^3 + 6n^2) =$ $\sum_{n=1}^{5} 4n^3 + \sum_{n=1}^{5} 6n^2 = 4\frac{5^2(5+1)^2}{4} + 6\frac{5(5+1)(2(5)+1)}{6} = 900 + 330 = 1230,$ D.

10. **A:** A tetrahedral die has sides from 1-4, meaning that the expected value is just the average of the low and high values since a tetrahedron is regular and every side has the same chance of being rolled. $\frac{1+4}{2} = 2.5$. Similarly, for a hexahedral die with sides numbered from 1-6, $\frac{1+6}{2} = 3.5$. And an octahedral die with sides numbered from 1-8, $\frac{1+8}{2} = 4.5$. A dodecahedral die with sides numbered from 1-12, $\frac{1+12}{2} = 6.5$. An icosahedral die with sides numbered from 1-20, $\frac{1+20}{2} =$ 10.5. And to get the expected sum of the values, we can just add all of these numbers, 2.5+3.5+4.5+6.5+10.5=27.5, A.

11. **C:** We could calculate an equation for f, but that's a bit more tedious than what we need to do. Instead, if we use the method of differences,

-28		-60		-8		134		372
	-32		52		142		238	
		84		90		96		
			6		6			

Since we know that f is a cubic, we only have to take differences down 3 levels, and once we've gotten to that row, we can just repeat the number and reverse our method all the way back up to get 372, C.

12. **D:** Rather than calculate x directly and try to take that value to the fourth power, we can just take both sides of the original equation to the fourth power.

$$(x + \frac{1}{x})^4 = 4^4$$

$$x^4 + 4x^2 + 6 + 4\frac{1}{x^2} + \frac{1}{x^4} = 256 \quad (1)$$

We can also solve for $x^2 + \frac{1}{x^2}$ by squaring instead of taking to the fourth power.

$$(x+\frac{1}{x})^2 = 4^2$$

$$x^2 + 2 + \frac{1}{x^2} = 16$$

$$x^2 + \frac{1}{x^2} = 14$$

Substituting back into equation (1), we get

$$x^4 + \frac{1}{x^4} + 4x^2 + 4\frac{1}{x^2} + 6 = 256$$

$$x^4 + \frac{1}{x^4} + 4(14) + 6 = 256$$

$$x^4 + \frac{1}{x^4} = 194$$

So we get our answer to be 194, D.

13. **A:** We start off noticing that in this problem, 7, 13, and 37 are factored in the denominator. Each of these are factors of 999,999. $7 * 11 * 13 = 1001$, and $3^3 * 37 = 999$. Knowing this makes the problem much easier, as we can now just multiply both the numerator and denominator by $3^3 * 11 = 297$. At this point, we have the fraction $\frac{594}{999{,}999}$. This will give us a 6-digit repeating decimal, $0.\overline{000594}$, A.

14. **B:** In order for each digit to be strictly greater than the one on its right, we have to have 4 different digits. If we choose 4 different from the 10 available, there is exactly one way in which we can order those digits such that they are in increasing order from left to right. Therefore, our answer is just $\binom{10}{4}$ = 210, B

15. **E, 9580:** Similar to problem 14, there are 210 ways to be strictly increasing order. There are also exactly 210 ways to be in strictly decreasing order since we can take each number in strictly increasing order and reverse the order of the digits to be in strictly decreasing order. This means that of the 10000 possible inputs, 420 of them are invalid. 10,000-420=9,580, E.

16. **B:** First, we need to calculate the area of the triangle so we can use $A = rs$. To calculate the area, we use Hero's formula. $s = \frac{8+10+12}{2} = 15$. $A = \sqrt{(15)(15-12)(15-10)(15-8)} = 15\sqrt{7}$. Now that we have our area, $15\sqrt{7} = r(15)$. So, $r = \sqrt{7}$, B.

17. **A:** Our numerator can be split directly down the halfway point. This gives us $(x^2y - 3x^2 - 7xy + 21x) + (xy^2 - 3xy - 7y^2 + 21y)$, factoring x out of the first part and y from the second part, we get $x(xy - 3x - 7y + 21) + y(xy - 3x - 7x + 21)$. This allows us to simplify our equation to $xy - 3x - 7y + 21 = 72$. We can use Simon's Favorite Factoring Trick on the left hand side to get $(x-7)(y-3) = 72$. At this point, the smallest pairing of

factors of 72 that allow *x* and *y* to be positive is 13,15. Using these two values, the smallest possible sum is 28, B.

18. **D:** Using the relation $\frac{||\boldsymbol{a}\times\boldsymbol{b}||}{||\boldsymbol{a}||||\boldsymbol{b}||} = \sin(\theta)$, we know that the sine of any angle must be between -1 and 1 inclusive, meaning that $0 \le ||\boldsymbol{a} \times \boldsymbol{b}|| \le \left|\left|\boldsymbol{a}\right|\right|||\boldsymbol{b}||$, or $0 \le ||\boldsymbol{a} \times \boldsymbol{b}|| \le 60$. Therefore, the number of integer values that $||\boldsymbol{a} \times \boldsymbol{b}||$ could take on is 61, D.

19. **E, $\boldsymbol{y^2 - x^2 = 1}$:**

20. **B:** To get from (0,0) to (4,6), we need to make 10 moves, 4 right and 6 up. To order these, we simply compute $\binom{10}{4}$ since we choose 4 of our 10 moves to be right. This gives us 210 total ways to get from (0,0) to (4,6). However, we cannot go through the space (2,3). There are $\binom{5}{2}$ ways to get from (0,0) to (2,3) and $\binom{5}{2}$ ways to get from (2,3) to (4,6). This gives us $\binom{5}{2}\binom{5}{2}$ paths that go through (2,3). $\binom{5}{2} = 10$, so we have $210 - 100 = 110$ paths from (0,0) to (4,6) that don't go through (2,3), B.

21. **C:** To go from (4,6) back to (2,3), we need to go left twice and down thrice. Since we can also move diagonally, we can replace one left and one down with a diagonal move. This gives us three cases:

Case1: No diagonals: 2 lefts, 3 downs gives us $\binom{5}{2} = 10$

Case2: 1 diagonal: 1 diagonal, 1 left, 2 downs gives us $\binom{4}{2}\binom{2}{1} = 12$.

Case3: 2 diagonals: 2 diagonals, 1 down gives us $\binom{3}{2} = 3$.

This gives us a total of 10+12+3=25 paths, C.

22. **D:** We know that if Elliot rolls a 9, 10, 11, or 12, then he automatically wins. This gives us a probability of $\frac{1}{3}$ immediately. Otherwise, we can compare Elliot and Jennifer fairly. If Elliot rolls between a 1 and an 8, which he does with $\frac{2}{3}$, there is a $\frac{1}{8}$ chance that he and Jennifer roll the same number. The rest of the probability can be split evenly as for every way that Jennifer can roll a higher number than Elliot, Elliot has the exactly same chance to roll a higher number than Jennifer. For example, Jennifer could roll a 5 while Elliot rolls a 3, but Elliot could similarly roll a 5 while Jennifer rolls a 3. Thus, $\frac{1}{8} + P(Elliot\ rolls\ higher) + P(Jennifer\ rolls\ higher) = 1$. But since $P(Elliot\ rolls\ higher) = P(Jennifer\ rolls\ higher)$, we can calculate $P(Elliot\ rolls\ higher) = \frac{1-\frac{1}{8}}{2} = \frac{7}{16}$. Therefore, Elliot's chance to win is $\frac{1}{3} + \left(\frac{2}{3}\right)\left(\frac{7}{16}\right) = \frac{15}{24} = \frac{5}{8}$, D.

23. **C:** We know that the product of all the roots, $abcd = \frac{396}{1}$ by Vieta's formulas. The sum of the product of the roots taken three at a time, $abc + abd + acd + bcd = -\frac{-11}{1}$. Dividing the product of the roots taken three at a time by the product of all of the roots gives us $\frac{abc+abd+acd+bcd}{abcd} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = \frac{1}{36}$, C.

24. **B:** First things first, there is only one way for Rickie to climb a single step. To get to the second step, Rickie could take 2 different paths: 2 single steps, one double step. From here, for each step, you can reach it by taking a double step from two steps below, or a single step from the step directly below. This gives us the relation $S_n = S_{n-1} + S_{n-2}$. This is just the Fibonacci recursive formula! We start with 1, 2, 3…, so the tenth number we reach using this relation is 89, B.

25. **B:** If we know that two chords are perpendicular to each other in a circle and one is a perpendicular bisector of the other, one of the chords must be the diameter. If the diameter were 12, there is no way that we could have a chord of length 16 in the same circle, so the diameter of our circle much be 16, which means the area is 64π, B.

26. **D:** In polar form, we can write conic sections by using the equation $r = \frac{ep}{1 \pm e\cos(\theta)}$, or $r = \frac{ep}{1 \pm e\sin(\theta)}$, where *e* is the eccentricity of the conic section and *p* is the distance from pole to directrix. In this case, we don't have a 1 as our constant in our denominator, so we divide out a 2 from the numerator and denominator. This gives us $r = \frac{2}{1-2.5\cos(\theta)}$. A conic section with an eccentricity of 2.5 must be a hyperbola, D.

27. **B:** First, we can simplify our product by taking out the $2^{12}5^{12}$ and replacing it with 10^{12}. Next, we know that taking the log of a number allows us to count the number of digits by taking the ceiling of the given number. $\log(3^{13}) = 13\log(3) = 13(.477) = 6.201$. Taking the ceiling of this gives us 7 digits. Now we add in the 12 zeroes added by the 10^{12} factor to get 19 digits, B.

28. **D (Thrown out):** At first, this problem looks like just a bunch of addition with not a lot of other information. Let's start by writing out some of the numbers that we might see.

1 1 1 1 1 1 1 1 1 1

1 2 3 4 5 6 7 8 9 10

1 3 6 10 15 21 28 36 45 55

We're starting to see some patterns with some recognizable numbers. It becomes even more clear what we're looking for if we turn things a bit.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

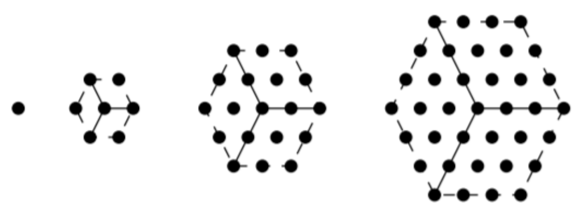
.

.

.

Aha! We see Pascals' triangle. So now, we just need to figure out what numbers Fletcher is calling out. He starts with 1, then 5, and so on. Since we're calling down diagonals, ie Alec calls out $\binom{n}{0}$, Bryan calls out $\binom{n+1}{1}$, etc., we can extrapolate this to find that Fletcher is calling out numbers, with his first one being $\binom{4}{4}$, second being $\binom{5}{4}$, and so on. This means that Fletcher's numbers can be generally written as $\binom{n+3}{4}$. Since Alec calls out 10 numbers, everyone calls out 10 numbers. If we let n=10 in our formula for Fletcher's numbers, we get $\binom{13}{4} = 715$, D.

29. **D:** Hex numbers have an interesting property that the sum of the first n hex numbers is equal to n^3. This can be seen by imagining each hexagon as 3 sides of a cube and seeing each additional hex number as a kind of sheath that can envelope the previous cube.



Alternatively, simply counting the dots in each of the hex numbers already given and keeping a running total allows us to see the pattern easily with the first few iterations of partial sums going 1, 8, 27, 64.

30. **B:** Trying to simplify the denominator or cross multiply it out actually makes this problem more difficult. Instead, recall our tangent double angle formula $\tan(2\theta) = \frac{2\tan(\theta)}{1-(\tan(\theta))^2}$. Now back to our equation, let's start simplifying by letting $y = \frac{2x}{1-x^2}$. Now we can simplify the left hand side into $\frac{2y}{1-y^2}$. If we let $x = \tan(\theta)$, then $y = \tan(2\theta)$, and $\sqrt{3} = \tan(4\theta)$. From here, we need to recall that $\sqrt{3} = \tan(60)$. Now, we have $\tan(4\theta) = \tan(60)$. Normally, if this were sine or cosine, we would add $\frac{360}{4} = 90$, but tangent has a period of 180, so we only need to add $\frac{180}{4} = 45$ to our original solution 3 times. Our unique solutions are $\tan(15), \tan(60), \tan(105), \tan(150)$, which can be evaluated to give us our solutions:

$2-\sqrt{3}, \sqrt{3}, -2-\sqrt{3}, -\frac{1}{\sqrt{3}}$. The positive solutions are $2-\sqrt{3}$ and $\sqrt{3}$. Of these, $\sqrt{3}$ has greater magnitude. The negative solutions are $-2-\sqrt{3}$ and $-\frac{1}{\sqrt{3}}$. Of these, $-2-\sqrt{3}$ has the greater magnitude. The sum, $-2-\sqrt{3}+\sqrt{3}=-2$, B.